

30th European Mathematical Psychology Group Meeting
Mannheim
Aug. 30 – Sept. 2, 1999

Abstracts

Signal Detection Theory: Fundamental Misrepresentations of Human Performance

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Signal detection theory statistics (e.g., d' and β) are often used to confirm or disconfirm hypotheses about the effects of experimental variables on 'sensitivity' while controlling for effects of 'response bias'. This attempt to add rigor to an hypothesis test is legitimate only under some assumptions (e. g. normality of the encoding distributions). However, many researchers have been taught to believe that these assumptions are either valid or harmless. Recently, I developed some new empirical tests of the signal detection theory model that do not depend on any distributional assumptions (Balakrishnan, 1998a; Balakrishnan, 1998b; Balakrishnan, in press). The results of these tests consistently violated the fundamental predictions of signal detection theory about the effects of bias on the placement of the decision criterion and the independence of encoding distributions and bias. In this paper, I review these findings and discuss their implications for human performance assessment. I then discuss the 'robustness' of the new tests to potential modifications of signal detection theory that might be proposed to explain these empirical results without dropping the concept of a variable detection criterion. Criterial noise, averaging of data across subjects and/or sessions, and sequential effects cannot account for the violations of the model's predictions. Finally, I propose some new measures and hypothesis testing rules that can replace the signal detection theory indices.

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On the Apparent Failure of the Strategy of Model Competition to Achieve “Final Theory” in Cognitive Psychology

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This paper briefly reviews the evolution of mathematical models in the more cognitive areas of memory, classification, choice response time etc. That is to say, areas of cognition that try to model categorical responses and/or choice response times, possibly with confidence judgments as well, but with no ties to the natural sciences like biology or physics. In particular, I will argue that the efforts to achieve “final theory” in the cognitive areas by following a process of competition among models have not worked. The reason it has not worked is that, at least without principled restrictions, there are too many ways to fit data, not too few. Participating in model competition involving data fitting and model falsification can be interesting and can require great skill, but it has not generated many, if any long term successes. I will illustrate the point by starting with the simple Bradley-Terry-Luce model of paired comparisons, justify a series of statistical and modeling moves to turn it into a theory for choice response time, and use it to “fit” most of the known properties of choice reaction time. I will conclude with what I think does emerge constructively from the evolution of cognitive theory by model competition.

Heuristic of Relevant Features and Perception of Randomness

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Experimentation during the last few decades has repeatedly underlined the presence of a consistent gap between man and machine (fed by classical probabilistic and statistical principles) in tasks of recognition and generation of random stimuli.

We develop an heuristic of relevant features aimed at explaining some salient aspects of this man/machine discrepancy. It proposes in essence that the perceptual probability $p(x)$ of stimulus x , as inferred by a human subject, can be estimated from the corresponding mathematical probability $P(x)$ as

$$p(x) = \sum_{x \in [x]} P(x)$$

where $[x]$ denotes the equivalence class of all stimuli z exhibiting exactly the same relevant features $\{C\}$ as x does. Equivalently, two stimuli possibly differing by details but identical with respect to their relevant features correspond to the same percept in first approximation. For instance, stimuli $x_1 = 110101$ and $x_2 = 111111$ have the same mathematical (supposedly uniform) probability $P(x_1) = P(x_2) = 2^{-6} = 0.016$, but different perceptual probabilities $p(x_1) = 62/64 = 0.97$ and $p(x_2) = 2/64 = 0.03$ when assuming $C = \text{"constancy / non-constancy"}$ to be the unique relevant feature.

In the statistical-like description of information processing (such as exemplified by signal detection theory, but applied here to discrete rather than continuous signal), the way man extracts percepts $[x]$ (models) from stimuli x (data) betrays the presence of post hoc testing and multiple comparisons in perceptual decisions. Finite range effects open up the possibility of simulating sequences perceived as random by penalizing the apparition of patterned words.

Models of Visual Attention: A Selective Review

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A selective review of formal models of visual attention is presented. The first part of the review describes the development of serial processing models from the simple serial scanning models proposed by Sperling, Estes, and others in the 1960s to selective serial models such as Guided Search (Wolfe, 1994). The next part reviews the development of parallel processing models from the independent channels models proposed by Eriksen in the 1960's to limited-capacity models and race-based models of selection. The last part focuses on my attempt to develop a unified theory of visual recognition and attentional selection (TVA; Bundesen, 1990, 1998).

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**Switching Attention:
Diffusion Models of Response Time and Choice
Probability for
Multiattribute Stimuli**

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Sequential sampling models are an attractive approach for modeling information processing in various psychological tasks. They have been applied to account for response times and choice probabilities. The approach presented here assumes that for multiattribute stimuli a different information accumulation process, described as a diffusion process, takes place for each attribute or dimension of the stimulus. Moreover, it is assumed that attention switches from one dimension to the next within the time course of processing. The question is how these single processes are combined to form one information accumulation process. Two types of models are proposed to answer this question: The first one assumes temporal additivity across attributes and the second assumes probabilities of switching among attributes. The models predict first passage time probabilities, related to observed relative frequencies and first passage times, related to observed mean response times. Here, the distributions of the first passage time are emphasized and it is shown that the models can be distinguished on the level of distributions but not on the level of means. Moreover, it is shown that the models not only predict faster answers to correct responses than to errors but also that they predict faster responses to errors than to correct answers simply by reversing the order of attended dimensions.

**The Nonparametric Procedure for Comparison of
Psychometric Functions**

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Application of the Method of Constant Stimuli (MCS) in psychophysical investigations implies subjects' responses to be statistically independent and stable. Actually a subject's decision making strategy changes during the course of experiment, and his or her responses may depend statistically on each other. Then the statistical estimates calculated as the average of measured values would be not correct. Nevertheless, special statistical procedures intended for testing MCS data reliability are usually not employed, and measurement errors are most often ignored. A nonparametric procedure is suggested to test whether results obtained in different experimental sessions (or under varied conditions) are statistically the same. The null hypothesis is that the probabilities of a subject's 'Yes' responses are equal in all experiments for given stimulus magnitude, for which data may be presented as a contingency table. The decision is based on the value of chi-square statistic for a set of these tables. The additivity property of the chi-square distribution is used to compare data of MCS experiments as a whole. Thus, the overall measure of discrepancy for all tables simultaneously is the sum of the respective measures of each table. However this test cannot be applied to experimental data immediately: chi-square statistic is suitable only on specified conditions. So in some cases results of measurements for adjacent stimuli should be combined, pooling the data to form a single table for a set of stimuli close in magnitude. Unfortunately there is no general solution with regard to what grouping rule is optimum, but the pooling categories may significantly affect the inference and its interpretation. So, an algorithm was designed to assess influence of grouping on the estimate. We illustrate this approach by analyzing results of a large set of MCS experiments on discrimination of the lines lengths. The technique was used to test two hypotheses: Test 1: Whether results of a set of experiments, conducted independently of each other may be considered as statistically equivalent. Test 2: Whether the subject's response depends statistically on his preceding decisions. To do this all set of experimental data should be split into two subsets depending of the subject's preceding response. Then these subsets can be compared as different 'experiments'. In general, the method of splitting a data set could be employed to verify a variety of hypothesis about subject's strategies. Exploratory analysis of primary data using a developed algorithm has revealed some peculiarities that are usually not detected by an ordinary statistical data processing, but may have effect on final results. As it would be expected, at least one of assumptions mentioned above is not supported for considerable part of

untrained subjects. They often use a strategy of orientation to preceding response, but a few of them apply this strategy regularly.

Systems for Adaptive, Computerbased Learning

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Systems for adaptive computerbased learning in the field of School-Mathematics are now on the market or available for research purposes. Such systems consist of several components, as, for example,

- the adaptive assessment component,
- the component for selecting problems and exercises adaptively,
- the component for querying experts on the structure of the knowledge considered,
- the component for integrating judgments of different experts, and
- the component for validating knowledge structures.

This paper discusses the theoretical foundations for these components, their implementation, their state of development, and ideas for further improvements.

Visual Space

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Sensory spaces are vector space representations of judgments given by subjects in perceptual experiments. Visual spaces are representations which pertain to spatial judgments as might occur in the study of orientation in three-dimensional physical space. A representation of these judgments by an analytical geometry provides an theoretical structure, in which the mapping can be interpreted as motion. Motions are automorphisms the invariances of which manifest themselves as properties of form, e. g. as a metric. In mathematics the transition from a synthetic geometry of points, lines etc. to an analytical geometry pertaining to tuples of numbers is achieved by coordinatization. The resulting numerical representation is based on qualitative restrictions which are imposed by compasses and ruler in synthetic geometry. A psychological representation, correspondingly bases the analytic geometry on spatial judgments of subjects in the visual laboratory. The paper will report different proposed solutions of this problem for binocular and monocular space perception as well as color perception.

Empirical Evaluation of Axioms Fundamental to Stevens' Ratio-Scaling Approach

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Stevens' direct scaling methods rest on the assumption that subjects are capable of reporting or producing ratios of sensation magnitudes. Only recently, however, an axiomatization proposed by Narens [J. Math. Psych., 40, 109-129 (1996)] specified necessary conditions for this assumption that may be put to an empirical test. In the present investigation, the central axioms of commutativity and multiplicativity were evaluated by having subjects produce loudness ratios. It turned out that the adjustments were consistent with the commutativity condition; multiplicativity (the fact that consecutive doubling and tripling of loudness should be equivalent to making the starting intensity six times as loud), however, was violated in a significant number of cases. According to Narens' axiomatization, this outcome implies that though in principle, a ratio scale of loudness exists, the numbers used by subjects in order to describe sensation ratios may not be taken at face value.

Media Theory**Jean-Claude Falmagne**

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Many situations where individual preferences change over time can be analyzed from the standpoint of a particular semigroup of transformations on a finite set of states. We require this semigroup to be transitive: any state can be transformed into any other state. Other axioms ensure the consistency of the transformations producing a state. The resulting semigroup is called a 'medium'. Applications of media theory are many and diverse, ranging from convex analysis to combinatorics and political sciences. For example, the family of all strict partial orders on a finite set, equipped with the set of transformations consisting in adding (or removing) an ordered pair to (or from) a partial order to form another partial order is an instance of a medium. We introduce the concepts of 'orientation' and of 'closure' for a medium and derive some consequences. An application of media theory to the analysis of opinion polls data is briefly discussed.

Mathematical Psychology and Psychophysics**Jean-Claude Falmagne**

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The impact of mathematical psychology on the theory and practice of psychophysics was via three topics: measurement theory, stochastic processes, and functional equations. The talk will briefly review the role of each, with a special emphasis on the latter. Some recent functional equation results concerning the so-called Fechner-Thurstone and Gain Control representations, in the framework of a near-miss-to-Weber's Law equation, will be described.

Simulation and Empirical Studies of Forced-Choice Staircases with Fixed Step Sizes**Miguel A. Garcia-Perez**

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Adaptive staircases are widely used in psychophysical research. To obtain a threshold estimate for some given stimulus, the conventional practice is to interweave a number of identical staircases each of which provides an estimate of the target percent point under the psychometric function; all of these individual estimates are then averaged to obtain a final threshold estimate. To reduce experimental cost, each of the interwoven staircases runs for very few trials (or reversals). This work assessed the comparative efficacy of an alternative practice, namely, running a single staircase for an equivalent number of reversals. A simulation study was conducted which showed that the distribution of the average of estimates from three interwoven, 14-reversal staircases has virtually the same mean as that of the single estimate obtained from a 42-reversal staircase, but the latter distribution is somewhat narrower. In addition, the three-staircase procedure always required a substantially larger number of trials than the single-staircase approach. Overall, these simulations (which yielded the same results for the 1-down/1-up (1/1), 2/1, 3/1 and 4/1 rules) indicate that a single, long staircase provides equally unbiased but more consistent estimates than the conventional multi-staircase approach, also incurring fewer trials and, thus, reducing experimental cost. An empirical study using actual human subjects in a detection task yielded results analogous to those obtained in the simulations.

Temporal Dynamics of Adaptation in Color Appearance and Discrimination

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Adaptation to a steady background has a profound effect on both color appearance and discriminability. The goal of these experiments was to determine the temporal characteristics of adaptation for appearance and discrimination, and for changes along different color directions.

Subjects were adapted to a large uniform background made up of a CRT screen and a $60 \times 60^\circ$ wall illuminated by computer controlled lamps. After an instant change in background color along the red-green or blue-yellow cardinal color axes, we measured thresholds for the detection of increments or decrements along the same axes at fixed times between 16 ms und 120 s. Analogously, color appearance was determined using achromatic matching.

A slow exponential time course of adaptation was observed with a half-life of 20–30 seconds that was common to appearance and discrimination. Also, a 50–100 ms component could be indentified, which was probably due to photoreceptor adaptation. There was an extremely fast mechanism with a half life faster than 10 ms, but only for color appearance. There were no differences for adaptational changes along the different color axes.

It can be concluded that the fast adaptation mechanism for color appearance is of higher order and situated after the mechanisms mediating slower adaptational changes in color discrimination and appearance.

On the Illumination-Invariance of Plateau's Midgrey

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In 1872, M.J. Plateau provided each of eight artists with a white and a black disk and instructed them to return to their respective studios and paint a grey disk in appearance midway between the two. Although the illumination conditions may have differed substantially across the studios, the resulting disks were virtually identical. Surprisingly, this remarkable result plays no significant role in current research on the perception of achromatic colors. Experimental as well as theoretical work nearly exclusively relies on cross-context indifference, but does not make use of a midgrey operation. On the other hand, J.-C. Falmagne has developed a formalization in which the invariance reported by Plateau leads to a specification of the form of the psychophysical function. The theory, however, is based on an assumption which may be valid in complex scenes, but usually is contradicted by experimental data. It is shown that Falmagne's approach can be generalized to apply to standard experimental situations. Within the presented formal framework, the impact of illumination-invariance on the possible form of the psychophysical function can be determined via functional equation techniques. In an experimental study, using decremental stimuli in simple center-surround configurations, the illumination-invariance of the midgrey operation turns out to hold for 5 out of 12 subjects. To account for the observed deviations we suggest a more general condition, called near-miss-to-illumination-invariance, which receives support from re-analysing the data of a classical experiment.

Probabilistic Assessment of Knowledge

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Various procedures for the consideration of noise (e.g. careless errors or lucky guesses) in the assessment of knowledge within knowledge space theory have been suggested. These procedures can be divided into two basic classes, discrete procedures which try to cover noise by correcting a previously made (preliminary) assessment error, and continuous procedures which estimate certain parameters of the probability distribution of knowledge states. In a simulation study, the different procedures are compared. Special attention is given to the robustness against noise.

How to Produce Fast Absent-Responses in Visual Search

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In visual-search experiments, participants have to decide whether a target item is present among distractors or not. Usually, target-present responses are faster than target-absent responses. However, when the distractors were homogenous and regularly arranged, then there was often an absent-advantage, i. e. absent-responses were faster. Up to now it is open what exactly determines this effect. It has been speculated that an absent-advantage occurs when the distractors can be grouped and rejected as a whole. However, also decisional processes might play a role. To test these and other hypotheses, we conducted a series of experiments. It turned out that several factors are necessary for an absent-advantage to occur. Absent-responses were faster than present-responses only when (1) the distractors were homogeneous, (2) the distractors were arranged as a regular pattern, (3) regular patterns were mixed with irregular ones across trials, and (4) the distractors were familiar. These results suggest that the absent-advantage is not due to perceptual processes alone, but depends also on decisional processes.

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Visual Space under Non-Egocentric Localization

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Traditionally, experiments related to the Luneburg's model have been performed with a configuration of stimulus points presented in a frameless space where the localization is totally ego-centric (Indow, 1991). It is because that all the assumptions in the model are not expected to hold under more natural condition. The basic assumptions are reduced to following two. In most experiments, the two have been treated as inseparable. A) Visual space (VS) is a hyperbolic space with a constant curvature K (< 0). B) The correspondence between VS and the physical space (X) is of a simple and rigid form. Evidently, the form of correspondence between VS and X is susceptible to change of condition in X . Hence, it is desirable to take a direct approach to the assumption A, keeping it strictly separate from the assumption B. Zimmer (1998) designed such an experiment with light points presented in the dark and showed that a most fundamental postulate for a space to be geometrical, betweenness, is clearly violated in some subjects. Because VS is the final product of a self-organizing process due to a given stimulus configuration, VS is dynamic and highly context-dependent. It is especially so in a frameless space. The difference between piecemeal tests of individual axioms and direct approaches to VS as a united entity will be discussed. A real world scene we see is a product of multiple glances of a complex stimulus configuration. Nevertheless, it is highly stable in a given context. Without referring to the assumption B, I discussed its geometrical structure at the EMPG meeting in Padoa (Indow, 1996). A perceived scene is a 3-D structure bounded by the ground and sky in which every percept appears at a finite distance from the subject. It is an interesting question to ask how the ground on which we are standing appears to be extended, if not obstructed, up to the horizon (the boundary of VS) or how the perceptual distance to the boundary is determined. Drösler (1992) discussed the ground and horizon in monocular vision. Indow (1997) discussed the boundary of frameless VS , within the framework of the assumption B, as the asymptote where the convergence vanishes. How far the horizon in a natural scene appears cannot be discussed in this way. The roles of vertical horopter, texture gradient, and optical flow will be discussed.

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**The Spherical Model of Color Discrimination
Based on Human VEP Data**

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The study of color discrimination in humans using multidimensional scaling of large color differences has revealed that construction of uniform color space in which Euclidean distances between color points will strictly correspond to perceived color differences is possible when light stimuli varying on spectral distributions and luminance can be represented as a points on a surface of a hypersphere in four-dimensional Euclidean space. Three spherical coordinates of the color hypersphere match subjective attributes of colors: hue, saturation, and brightness, while four Cartesian coordinates represent four neuronal channels (red-green, blue-yellow, bright, and dark) of the visual system. A subset of equibright colors forms three-dimensional Euclidean subspace in which color points are located on a two-dimensional spherical surface. (Izmailov and Sokolov, 1984, 1991; Izmailov, 1995). Color functions calculated from spherical model are in good coincide with psychophysical and neurophysiological data of color vision (Izmailov, 1980, Izmailov, Sokolov, Chernorizov, 1989). Cortical evoked potentials in human were studied to obtain direct electrophysiological measures of color differences and to compare them with color discrimination data obtained by psychophysical methods. The analysis of evoked potentials was different from traditional experimental techniques by two basic peculiarities. Firstly, instead presentation of separate light flash, the instant (with front no more than 0.5 ????) substitution of one flash of light on the other was used. Evoked potential was recorded on this change (Paulus et. al., 1984; Zimzhev et. al., 1986). Secondly, the difference between changed stimuli in pair was increased monotonically till both legs from zero point, in which changed stimuli were identical. For the specification of color components series of such functionally connected evoked potentials on stimuli substitutions are used rather then separate evoked potentials (Zimzhev et. al., 1986, Izmailov et. al., 1998). Experiments with pair substitution of five color stimulus (four basic colors - red, blue, yellow, green - and white color) each varying on seven levels of luminance and 12 equibright colors were spend. A color coding component N87 and luminance coding component P120 were reliably found in occipital (O1,O2) and temporal (T5,T6) loci. (Paulus et. al., 1984; Regan, 1989). However, analysis of latency and amplitude of components N87 and P120 depending on differences of luminances between stimulus with identical and different spectral distributions in a pair has shown, that N87 reflects not only color, but also the brightness differences. It allows to connect N87, as one of earliest component with a N1 in cortical evoked potential

in primate, reflecting activity of cells, receiving the information directly from NGB and to consider it as the first stage of the cortical analysis of chromatical and achromatical characteristics of light (Padmos, Van Norren, 1975; Zrenner, 1983). Concerns to P120 traditionally interpreted as brightness component, it represents activity of cortical cells connected to the analysis of non-chromatic characteristics of stimulus, such as the form, movement, orientation and etc., which are also based on luminance gradient and contrast, but not directly connected with brightness of light. It explains long latency and high amplitude of P120, in comparison with N87, and also correlation P120 with the configurative characteristics of stimulus, that is referred in a number of works (Regan, 1989). For a comparison of the evoked potentials with subjective estimates of color differences three indexes were used: amplitudes of components N87 and P120 relatively to background and interpeaks amplitude (between N87 and P120). Scatter diagrams between subjective estimates of color differences and these indexes are drawn. The highest correlation with psychophysical estimates of color differences is shown by interpeaks amplitudes for temporal electrode site T5. Using the interpeaks amplitudes as measures of color differences for twelve stimuli color space had been constructed by multidimensional scaling technique. The color points were located on the spherical surface in three-dimensional Euclidean space in exactitude as well as in case of subjective estimating of these color differences.

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Photopigment Opsin Variation and Color Sensation

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Cognition and color appearance research frequently makes use of Trichromatic Theory in modeling higher-order color sensations. Recent results in molecular genetics indicate that many individuals express more than three color vision photopigments. To investigate the influences of more than three photoreceptor types on color sensation we assessed the color perception of individuals grouped by classes of photopigment opsin genes identified through Polymerase Chain Reaction (PCR) genetic assays. We compare groups of individuals identified as Dichromats and Trichromats with individuals exhibiting non-standard photopigment variations (including heterozygous and polymorphic variants). Two methods of assessing color sensations were used: (1) a method using a diffracted spectrum with heterochromatic luminance (Jameson, Highnote and Wasserman 1998), and (2) a method using an equal-energy spectrum (Bonnardel, 1998). Comparison of the results further clarifies the relations between photopigment opsin genotypes and color perception. Implications for cognitive models of color appearance phenomenology and Graßmann color coding are discussed.

Using Unforced-Choice Tasks in Adaptive Psychophysical Procedures

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Adaptive procedures are in widespread use for measuring absolute and differential thresholds. They can be combined with a variety of tasks, the most common ones being yes-no tasks and forced-choice tasks. Yes-no tasks are often considered to be problematic due to the necessity of the participant to maintain a criterion (where to start to say 'yes') at a stable position. Naive participants are often not able to do so, and there might be a considerable variation between participants concerning the position of this criterion. Forced-choice tasks circumvent this problem by having the participant compare two or more intervals. The adaptive rule is in general different from that of a yes-no task in order to account for chance performance at low signal levels. The latter gives rise to a random walk behavior of the adaptive track at low levels which reduces the reliability of the procedure and thus its efficiency.

The present talk presents a rule that may be combined with unforced-choice tasks. A theoretical analysis of unforced-choice tasks reveals that while the latter involve a criterion (where to start to say "don't know") the situation is nonetheless quite different from that of yes-no tasks. The effects of the criterion and of its possible variations on the value and standard deviation of the threshold estimate are shown to be of no importance as compared to the gain obtained by reducing the random walk behavior for low signal levels. The resulting procedure proves to be more efficient than yes-no and forced-choice procedures. This is shown with computer simulations as well as with empirical data. Moreover, participants report the new procedure to be much more comfortable. In summary, using unforced-choice tasks in adaptive procedures is preferable over using yes-no tasks or forced-choice tasks as it is slightly more efficient and much more comfortable. It might prove of special benefit in clinical settings where the participants are naive and one is in want of a comfortable, yet precise procedure.

Item Selection in IRT Models for Measuring Change

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Subject of this paper are certain topics concerning the measurement of change for categorical variables by Item Response Models. The experimental design used for this is the most simple design: a 2×2 -design with measurements at two different timepoints in two different groups. For measurement of change with categorical variables mainly two Item Response Models are proposed: The LLTM (Linear Logistic Test Model) and the LLRA (Linear Logistic model with Relaxed Assumptions). Both models were developed by G. H. Fischer. In both cases change is measured by using an existing group structure, assuming that inside of these groups the amount of change is the same for all members of the group. There can be a systematic error distorting the measurement, if a) the assumed group structure is wrong and a different group structure was convenient for measuring change, or if b) there does not exist any kind of group structure at all. In this case, there would be a parameter measuring change for every person. In literature this phenomenon is called modifiability. In this paper strategies are presented for selecting a set of items- resp. persons- minimizing this bias. These strategies include certain methods of correspondence analysis, and certain tests of fit for the Rasch model. It will be examined if these strategies are suitable for our purpose.

**Nail that Distribution:
Linear Weighted Moments for Identifying the
Form of Reaction Time Distributions Generated by
Perceptual, Cognitive and Decision Tasks**

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Identifying the form of reaction time distributions (gamma, lognormal, ex-Gaussian, normal mixtures etc.) is a hoary, but important problem in mathematical psychology. So far its solution has proved elusive (Luce, 1986). This is because current methods need 1000s of observations per distribution, are prone to statistical 'mimicry' problems (Van Zandt & Ratcliff, 1995), and can only combine data across people and treatments by introducing many additional (improbable) assumptions. Linear weighted moments based parameters (Hosking, 1990) (second linear moment for spread; linear skew for asymmetry; and linear kurtosis for high-tailedness) can overcome these difficulties. Theoretically, the relations among these parameters distinguish among statistical distributions - as with conventional methods. However with linear weighted moment methods sampling variance is low enough to avoid mimicry problems, and sampling bias is low enough to allow combining distributions with minimal (testable) assumptions. The methods are illustrated with data from signal detection, mental arithmetic, sequential keying and visual search.

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Discrimination of Depth and Criterion-Setting

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Do cognitive factors affect the judgment of apparent depth in standard psychophysical discrimination tasks? To examine contextual effects on discrimination performance, we conducted two experiments. In each the MSS was used and contextual effects were produced by varying the location of the range of test stimuli. Ocular vergence served as the only depth cue in the first and horizontal disparity in the second experiment. In each trial of the first experiment a fixation cross flanked by nonius lines was presented in a Wheatstone configuration for 1.4 sec before it disappeared. After a short interval, a vertical bar was presented in an extended dark field and the subject verged on the stimulus. The bar remained present until the subject had responded. Each subject judged if the bar appeared to be in front of or behind the previously seen fixation cross. In three sessions the midpoint of the test stimuli was centered on -2 , 0 , or $+2$ arcmin from the fixation cross. The second experiment was conducted in a dimly lit room where the monitor face provided a fusion lock. On each trial the fixation cross remained visible throughout so that the subjects could base their judgments on the horizontal disparity of the bar. In three sessions the midpoint of the test stimuli was centered on -2 , 0 , or $+2$ arbitrary units (AU) from the fixation cross. (A single AU was half the individual discrimination threshold as determined by an adaptive staircase procedure before the first test block. The results show that ocular vergence is an unreliable depth cue that generates uncertainty and promotes contextual effects. On average the discrimination thresholds were twelve times higher than for horizontal disparity judgments. Nevertheless, the latter were also susceptible to contextual effects. The contextual effects for depth discrimination are similar to those that have previously been shown for spatial frequency and orientation discrimination, and can be explained by criterion-setting theory (Lages and Treisman, 1998). This supports the generality of criterion-setting effects in visual discrimination.

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**A Method for Approximating the Shape of Latent
Fields of
Single Visual Points for Predicting Geometric Optical
Illusions.**

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The length illusions within geometric figures, e. g. the Müller-Lyer figure, can be predicted to a certain degree by the additive superposition of the illusions caused by interactions among pairs of lines. We therefore focussed our investigations on basic experiments, where the subject had to reproduce a line S out of two random lines (S, G) . G had a constant length of 10 cm. The averaged length illusions of the reproductions of S of about 1300 stimuli (S, G) could be predicted with correlations of .60 – .80 by a two-parameter field model. It postulates an influence of G on S by a potential field around G resulting from the superposition of concentric fields emitted by the visual points on G . In the model the strength of the field in a point s on S emitted by a point g on G is defined by a two-parameter function $f = f(d(s, g))$ depending only on the distance $d(s, g)$. The theoretical length illusion $\theta(S, G)$ of S in the field of G then results from a double line integral of the function f over G and S :

$$\begin{aligned}\theta(S, G) &= \\ &= \int_S \int_G f(d(s, g)) dg ds \\ &= \int_{x_0}^{x_1} \int_{\xi_0}^{\xi_1} f \left(\sqrt{(x - \xi)^2 + (ax + b - \alpha \xi - \beta)^2} \right) \\ &\quad \cdot \sqrt{1 + \alpha^2} \sqrt{1 + a^2} d\xi dx\end{aligned}$$

with $S: ax + b$ and $G: \alpha \xi + \beta$. The two integrals postulate an infinitesimal additivity holding as well for the concentric fields of the points g on G as for the assumed local illusions in S . On the other side, by testing about 12 elementary two-parameter distance functions f , we found that for 4-5 different functions the model fitted the data almost equally well. Apparently, the data fulfilled the two additivity conditions to such a high degree, that the choice of the distance function f turned out to be of secondary importance. The problem was, how to determine the "true" distance function f from the data. The new result is, that we found a method which allows to estimate the shape of the latent function f from our experimental data. We approximate the two integrals of $\theta(S, G)$ by a weighted linear function $w = w(d(s, g))$ of the distances $d(s, g)$ of all pairs of points (s, g) within our stimuli (S, G) . The weight function w obtained from the data approximates the shape of the latent function $f(d(s, g))$. The shape turned out to have the form of a damped oscillation.

Moreover, we found very similar oscillatory functions f for three geometrically different subclasses of our stimuli. The oscillatory functions probably can be reduced to a neurological background. The next problem is to find a mathematical model predicting the obtained damped oscillation.

Measurement and Utility

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A brief summary is given of what I consider to have been the major accomplishments in axiomatic measurement theory (psychometrics and scaling excluded) during this century and of what seem to be some of the outstanding problems. Examples are provided of applications of some results to the study of decision making under certainty and uncertainty.

Levels of Measurement Laws

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Three levels of scientific laws that involve measures onto continua are distinguished. The first level formulates fundamental, qualitative properties of attributes. These involve orderings coupled with other structures, and they lead to numerical representations. The most common are extensive, intensive, and conjoint measurement, of which physics and psychology provide many examples. The second level consists of laws linking distinct structures of the first type. In physics, distribution laws give rise to the structure of physical quantities; in utility theory, somewhat similar laws link joint receipt to gambles; in psychophysics, comparable laws seem less common. The third level involves laws stated within frameworks arrived at the second level. Sometimes these take the form of invariance principles, as in physical dimensional analysis, or as dynamic systems, as in much classical physics. Invariance examples from psychology are examined and contrasted with those from physics.

Order Structure of Order Relations

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In empirical sciences order relations represent basic relationships such as being less-than or equal-to with respect to some attribute. Notions of progression, precedence, or preference are typical applications, most frequently used are equivalence relations, linear orders, partial orders and others. In mathematical psychology additional order relations with specific properties have been developed, including semi order, biorder, or interval order. This paper investigates logical relationships of the most prominent order relations within psychology. It is shown that the set of order structures is itself an interesting order. A diagram of its structure is presented and for every possible order type a minimal example is given.

Knowledge Representation and Cognitive Processes

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Incompleteness and Probability Modelling

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Probability models assume that uncertainties can be characterised by urns from which different types of ball are randomly drawn. This implies the existence of a veridical representation where uncertainty is expressed as a finite set of rational numbers. Probability models are often assumed to be complete i.e. to characterise all the uncertainty that is present by incorporating all distinctions affecting this uncertainty. This is not possible because of the following arguments.

1. Probabilities cannot represent total uncertainty.
2. Probabilities apply to classes of events rather than to unique events.
3. Probabilities are subjective as the classes they apply to must be defined by someone.
4. There are not enough rational numbers to distinguish between all conceivable classes of events.
5. Probabilities are affected only by pre-specified distinctions in a probability model.
6. Probabilities cannot represent the uncertainty associated with themselves.

Thus with any probability model there is uncertainty that it has characterised and uncertainty that it has not characterised. Evidence can affect both types with the result that Bayes theorem and the likelihood principle break down. The use of normative modelling in the evaluation of people's judgements of uncertainty is reassessed in the light of these considerations. Finally, an analogy is drawn between probability models and geometric figures, both of which force nature into mathematical structures which do not fit. Both are useful but, because of incompleteness, there is less reason for supposing that an uncertainty corresponds to an elaborate probability model than that a naturally occurring shape corresponds to a set of connected regular curves.

**An Alternative Framework for
Decision Theory in the Context of Multicriteria
Decision Aiding**

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Most theoretic and axiomatic works in the field of multicriteria decision aiding rely heavily on conjoint measurement. But, actually, conjoint measurement is not completely appropriate in this context. The main primitive in conjoint measurement is an overall preference relation. And one looks for conditions ensuring that this preference relation can be represented by a multidimensional structure.

In a decision aiding context, the overall preference relation does not exist. The decision maker doesn't know if he prefers x to y . The goal of the decision aiding process is precisely to build this preference relation.

We present a very general framework in which most decision aiding procedures can be described and we provide some characterizations in this framework.

**Adaptive Methods for Estimating Multivariate
Psychophysical Thresholds**

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When analyzing the integration of information from different sensory channels, the determination of thresholds for stimuli with more than one physical parameter (e. g. binaural stimuli) may be necessary. Like in the univariate case, adaptive procedures can be used to estimate thresholds effectively.

An example is the estimation of a combined stimulus (X, Y) with a given detection probability p subject to the constraint that the detection probabilities of the isolated stimuli $(X, 0)$ and $(0, Y)$ should be equal, i. e. the point (X, Y) of the psychometric function $f(x, y)$ with $f(X, Y) = p$ such that $f(X, 0) = f(0, Y)$ has to be determined.

Appropriate methods for this problem are multivariate Robbins-Monro-procedures, sequential methods on discrete lattices (generalized up-down methods) and an extension of Mukerjee's procedure. Additionally an algorithm for the isotonic regression problem on partial ordered sets is given, which is needed for Mukerjee's procedure and to get consistent estimators for up-down-methods.

A Preference Response Continuous Scale Model

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Wide range of models of preference are based on the so-called "single-peaked" paradigm, that is each subject's preference generally increases up to a point (the "ideal" point of the subject) in the ordering of the evaluated objects in question, and then decreases. An important class of such models is the parametric unidimensional unfolding model which is used mainly for scaling bipolar concepts. In this approach (real valued) positions in a latent dimension are attached to the subjects and the objects, and given these positions, the response probabilities of a subject are assumed to be a function of the distances between the positions of the subject and the objects. In our talk a new preference response model for scaling bipolar concepts will be suggested. The fundamental mechanism at work in preference formation is a kind of bisection of the opposite connotations of the bipolar concept in question. The derived apparent unidimensional model can be characterised by the subject and stimulus parameters, and the subjective weights of the opposite sides. Using simulated data, the estimation of the parameters are compared with the results from factor analysis and multidimensional scaling.

A Mathematical Theory of Belief

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A theory of belief is axiomatized in terms of a qualitative binary relation that may be interpreted as "the degree of belief of the conditional event $A|B$ is greater than the degree of belief of the conditional event $C|D$ ". Representation theorems establish that the qualitative axiomatization is a proper generalization of a theory of conditional probability. As a generalization, it has various interpretations, including one where uncertainty is measured in terms of two dimensions - one being probability (as usually defined) and the other a dimension called 'ambiguity'. There are strong links between this theory of belief and the descriptive theory of probability judgments known as "Support Theory".

The Development of Mathematical Psychology in the Future

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The closest perspective for mathematical psychology is the change of detail description to the discovery of fundamental laws. In theoretical psychology we will see the realising of the fact, that man is a temporal creature. An *apparatus criticus* will be worked out in mathematical psychology, and this *apparatus criticus* will be helpful for description of the dynamic of both psychical world reflection and world creation. The fundamental laws in mathematical form will be discovered.

MUEST: an Adaptive Psychometric Method

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MUEST is an extension of the so-called QUEST method of Watson and Pelli. As in QUEST, MUEST gives an Bayesian estimate of the "threshold" (offset of a psychometric curve), but additionally also estimates the steepness of the psychometric curve. After each trial, an optimal placement of the next stimulus level is calculated on basis of the threshold and steepness. The method and its advantages will be presented and some, previously time-consuming, applications will be discussed.

Quantitative Methods of Psychological Activity Theory

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The aim of this work is creation of psychological activity system general model with the help of sinergetic approach. Analysis of sinergetic models in psychology shows that the object of researches are: cognitive structures, memory, brain, intelligence, knowledge; mutual relations in small and big cooperations; evolution models; models of strategy choice and decision making; motivation models and etc. These parameters are the elements forming the system of activity in psychological conception of Leontev, Lomov and etc. First variants of activation models of two types were proposed: theoretical and experimental. In the theoretical model the activity is considered to be a function of variables: motivation, aims, needs, decision making and etc. For each variable the model specific parameters (points, circles or attracts), which defines the activity is built. The experimental model comes from the concrete activity. These stages of model creation were defined:

- definition of parameters of concrete activity,
- clearing of order parameters with the help of factor analysis,
- building the models allowing to define the stable condition of these parameters.

The model of activity of the managers of scientific-technic centre was worked as the experimental model. The following order parameters were defined: factor of ability to system intellect work and factor of decision making and responsibility for them. For these factors the regressive curves and LamereiÆs diagrams were constructed, which allowed to define stationary conditions. In the theoretical model the successful activity is considered to be a function of variables: motivation, individual intelligent, special intelligent, threshold level, characteristic of professional activity, individualities. If the individual intellectual level of the person is lower, than the 'threshold' than this person will not be successful in this activity. And if the individual intellectual level is higher, than the success of the person's activity will be determined by motivation, purposes, values, environment, etc. Preliminary examination of the model was carried out at the example of revealing the activity success of the managers in scientific - technical centre. Two types of successful managers were distinguished, and style factors of the success of the manager with different styles were discovered.

Where Process and Measurement Models Meet, Revisited

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Some years ago, Drösler (1981) has advocated to apply theories of measurement to problem solving, an area of psychology where - up to then - such models had hardly been applied to. With a somewhat different point of view, Gregg & Simon (1967) have compared mathematical theories to computer simulation models with respect to their costs and benefits. On the basis of these methodological investigations, I will present an overview of a contemporary cognitive architecture of human problem behavior (Anderson & Lebiere, 1998). I will thereby apply a levels description (Schmalhofer, 1998, chapter 3). A discussion about which aspects of current models of problem solving (e.g. Lifschitz, 1987, Anderson & Lebiere, 1998) satisfy the objectives of mathematical theories will conclude my presentation.

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A Method to Compare Knowledge Structures Concerning their Adequacy

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In practical applications of knowledge space theory we often have different knowledge structures for a knowledge domain Q . If we query, for example, two experts with the QUERY routine, they will produce in almost all cases different knowledge structures. If we derive knowledge structures from cognitive models of the problem solving behaviour, competing models will produce different knowledge structures. A method to compare these knowledge structures allows us to compare the underlying process models. We present a method which allows us to compare two knowledge structures concerning their ability to describe a set of observed response patterns from the domain. Simulation studies show that our method is highly accurate. The knowledge structure closest to the ætrueÆ knowledge structure is in almost all cases preferred by the method. But the results also show that the number of response patterns we need to guarantee reliable results increase strongly with the size of Q . Thus, the method is restricted in its practical application to small knowledge domains.

A Theory of Isoluminant Chromatic Motion Perception

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An isoluminant chromatic display contains different colors but all of the colors are precisely equal in brightness. Therefore, the motion in such displays is perceived neither by color-sensitive mechanisms of the brain, which are thought to be insensitive to motion, nor by motion-detection mechanisms, which have been shown to be insensitive to color. Four experiments are reported to demonstrate that isoluminant chromatic motion is perceived exclusively via the so-called third-order motion-detection system. [Note: The first-order motion system computes motion from the luminance $l(x, y, t)$ in a display. The second-order system computes motion from the quantity of texture $q(x, y, t)$ at each point of a display. The third-order system computes motion from the salience $s(x, y, t)$ of each point in a display. Salience is 1 for points that are segmented as figure, 0 for points segmented as ground, and salience takes intermediate values for ambiguous points.] The experimental results show that isoluminant chromatic motion has the following properties, all indicative of third-order motion: It fails the pedestal test, (2) has a temporal corner ("cutoff") frequency of 3–6 Hz, (3) is indifferent to whether successive frames are delivered to the same or to different eyes, and (4) the illusion of "motion standstill" can be produced with highly visible chromatic gratings that satisfy an equal-salience condition. These findings indicate that movement of pure-color displays is not detected by some subsystem of the brain's color-sensitive perceptual mechanism, but by the third-order motion system at a higher level of the brain where cues about form, texture, depth and color are simultaneously available and where selective attention can influence the process.

This work was supported by the U.S. Air Force Office of Scientific Research, Life Sciences, Visual Information Processing Program.

When Automorphisms are Rare

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The questions of scale type of a measurement structure are answered (partly) by investigating the automorphism group of the structure. The famous Alper & Narens Theorem characterizes automorphism groups on the reals. Recently, Suck (in press) generalized this result by developing a topological description based on the order topology of the measurement structure. This technique, as well as the ideas of the Alper/Narens Theorem, live on a rich automorphism group. What if this group is trivial, i.e., consisting only of the identity map? The present paper deals with possibilities to extend the topological approach to this situation. The tool employed is a groupoid instead of the automorphism group.

Suck, R. (in press) Scale type (N, N) and an order based topology induced on the automorphism group. *Journal of Mathematical Psychology*.

Learning

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A Boolean Approach to Hierarchical Data Analysis, an Overview

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Boolean analysis of questionnaires (Flament, 1976) is a method for data-analysis with a much larger scope than only questionnaires. In fact any set of binary data can be analyzed with this method and results in a unique set of equivalent implication schemes. Boolean analysis – we drop the ‘questionnaires’ from here on – is a partial order generalization of Guttman’s scalogram analysis. One of the criticisms to scalogram analysis is that in practice it is rare to find data that can be fitted well to the scalogram model. Generally other than the Guttman patterns are observed which results in low fit parameters. In Boolean analysis this problem does not exist, as this method basically does not impose any constraints on the patterns that can be modeled. At first sight this feature seems most promising: indeed, Boolean analysis allows one to build a perfectly fitting model for any set of observed patterns. In practical applications, however, approximated implication schemes that meet some criteria concerning avoidance of error, simplicity or structure are preferred over perfect schemes that accommodate all observed patterns. This paper provides an overview of methods, called dichotomization methods, that were introduced (see for example Theuns, 1994, 1998) for selecting ‘optimal’ sets of patterns to be modeled. Depending on the kind of application the selection method focuses on either avoiding error, simplicity of the resulting implication schemes or (closure) criteria concerning the structure of the set of modeled patterns.

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Assessing the Variability of Psychometric Functions

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The quality of psychophysical threshold measurements hinges on the availability of error estimates for the estimated parameters. To obtain the latter is particular difficult in work with patients and in situations where data are not stationary (e.g. due to learning). Bootstrapping (Efron & Tibshirani, 1993) provides a general and elegant way to estimate standard errors for arbitrarily complex summary statistics like those specifying the complete shape of a psychometric function, or, more generally, like function parameters in a nonlinear fit. The surprisingly simple idea of the bootstrap is to generate sets of virtual data by resampling (with replacement) the original data. In psychophysics the original data set are the responses collected in a session; the sets of resampled data have the same size as the original and consist of the same responses except that some responses have been omitted and others have been doubled. Using the Bayesian fit routine of Treutwein & Strasburger (1999) to fit a template psychometric function to the original data set, best estimates for threshold, slope, and guessing and lapsing rate are obtained, the variability of which are of interest. With the same fit routine applied to the sets of virtual data, distributions of these parameter values are obtained. The variances of these distributions are the ideal bootstrap estimates of the standard error of the corresponding parameters estimated from original set. Data from Monte-Carlo simulations demonstrate that the bootstrap distributions of the estimated parameter values are highly similar to the distributions of repeated (simulated) measurements. Data from psychophysical experiments further show the applicability of this method to real-world data.

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A Model for Speed and Accuracy Data on Complex Mental Tasks

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Since speed and accuracy form two complementary aspects of human performance, it seems interesting to develop a model that takes into account both aspects at the same time. For simple choice paradigms, this has been worked out by Ratcliff (e. g., Ratcliff, 1978; Ratcliff & Rouder, 1998). On the other hand, in the domain of complex mental tasks (or intelligence tests), such a model is less well developed. Models are either specified for accuracy data (e. g., Embretson, 1995) or reaction time data (e. g., Smit & Van der Ven, 1995), but seldom are time and accuracy modeled simultaneously. Two exceptions are the models of Verhelst, Verstralen, and Jansen (1996) and Roskam (1996), which write the probability of success as a function both of a set of basic parameters and reaction time on that particular item.

The present paper proposes a model that applies to complex mental problems (e. g., analogy solving). The model specifies both the time an examinee takes to finish an item and the probability of success on that item as a function of a set of common parameters. Generally speaking, the model states that an item is solved by the subsequent sampling of relevant solution principles until either the correct one is found (which results in success) or until a misleading rule is found (which results in an erroneous response). The model is applied to a test of solving number-series items and it is shown that the model is in good accord with these data. Although the present paper does not reflect explicitly on accomplishments of the past, it will be seen that the model incorporates trends in mathematical psychology like statistical learning theory (e. g., Estes, 1950) and speed/accuracy tradeoffs (e. g., Ratcliff, 1978).

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Accuracy, Response Time, Confidence and Short-Term Memory in Simple Decisions

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Most theoretical analyses of psychophysical decision processes have focused on accounting for response probabilities and times and have tended to overlook confidence assessments and to postpone the problem of transitory representation in memory. It is argued that, in addition to their intrinsic interest, confidence assessments present a challenge for theoretical models of psychophysical judgment designed primarily to account for tradeoffs between speed and accuracy. In a similar manner, the need to account for information loss in short-term memory substantially influences the pattern of predictions generated by stochastic decision models. Several hypotheses concerning the basis for confidence assessments and the representation of information in short-term memory are compared and evaluated. Results are presented from a series of experiments involving judgments of the relative frequency of two alternative stimulus elements within rapidly presented sequences. This task allows comparisons to be made between empirical data and theoretical predictions for each individual trial. The results of these comparisons are discussed in terms of competing models of the decision process, combined with alternative hypotheses regarding confidence and the operation of short-term memory. It is concluded that the data are best accounted for by an accumulator decision process, in conjunction with a balance-of-evidence hypothesis concerning confidence and a loose capacity model of short-term memory.

Non-Euclidean Property of Blind Walking**Toshio Watanabe**

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In the present study, the geometrical property of blind walking was studied. The subject was asked to make a circle of radius 2.3 m by blind walking. First, the blindfolded subject stood at the starting point O . Second, he walked 2.3 m subjectively and stopped (point A). Third, from point A , he drew a circle of radius \overline{OA} by walking. He was asked to put a small object for the record on the floor at the location of a third (point B), two third (point C) of the circumference and the final location (point D) of the circumference. Eight subjects attended the experiment and the experiment consisted of eight trials (four clockwise directions and four counter-clockwise). If the geometrical property of blind walking is Euclidean, points B , C , D should be on the same circumference as point A . However, the result showed that they were not on the same circumference and the average radius of \overline{OB} , \overline{OC} , \overline{OD} was significantly smaller than the radius \overline{OA} ($t=2.45$, $df=2$, $p < 0.05$). If we draw a circle of radius r on the surface of a globe, its circumference is smaller than $6.28r$. That is, the result implies that the geometrical property of blind walking should be elliptic.

Conceptual Knowledge Representation in Psychoanalytic Process Research**Karl Erich Wolff & Norbert Spangenberg**

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The purpose of this paper is to demonstrate an application of Conceptual Knowledge Representation in the field of psychoanalytic process research. Conceptual Knowledge Representation is based on Formal Concept Analysis introduced by R. Wille (1982). For a short introduction the reader is referred to Wolff (1994) and Wille (1997), for the mathematical foundations to Ganter, Wille (1999). As an example in psychoanalytic process research we use data of anorectic young women investigated by Spangenberg (1990). A central problem in the formal representation of processes is the determination of a suitable frame. Generalizing the usual film representation of a kinematic process we describe a psychoanalytic process by introducing a conceptual frame into which the given data can be embedded in a suitable granularity. This conceptual frame describes the language of a psychotherapist about those attributes which were used by the patient for the description of her family members. Combining the pairs (family member, time) as objects with the attributes of the psychotherapist we get a formal context whose concept lattice yields a visualization of the psychoanalytic process of the patient. This method was used by Wolff also for the description of industrial processes.

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Twelve Years of Modelling of Stereokinetic Phenomena

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Stereokinetic phenomena are 3-D percepts produced by 2-D figures rotating on the frontal plane. The mathematical model was based on a hypothesis of minimization of the velocity differences between the various points of the configuration. The main phenomena considered were: the apparent cone, produced by a rotating disk with an excentric dot inside it; the tilted disk and the ellipsoid, produced by a rotating ellipse; the tilted line segment, generated by a rotating line segment of constant length. Generalizing the model to periodic translational movements on the frontal plane we considered, among others, the swinging gate, produced by a vertical line periodically receding from and approaching a stationary vertical line of the same length. In all examined cases the theoretical predictions were in good agreement with the experimental results, obtained by asking the subjects to evaluate the apparent displacement in depth of the perceived images. The problem we are now facing is the apparent movement in depth of vertical line segments and of vertical rectangles of constant width, undergoing periodical contractions with simultaneous lateral displacements.

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**Riemannian Geometric Framework for Perceptual
Binding and
Object "Oneness"**

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Parallel processing of a visual attribute (such as motion) across locations require that those locations occupied by one object be segregated into a single region or "chunk" that reflects the "oneness" of that object. This grouping/binding is both dynamic, in the sense that it depends on the object's momentary location and the region of space it occupies, and automatic, in the sense that it depends only on the bottom-up sensory operations uniformly applied to all locations to start with. However, sensors that directly compute raw image features do not immediately provide such constancy. For example, the output of local motion sensors necessarily confound object velocity with image luminance structure (the so-called "aperture problem"), and therefore are not uniform across the region occupied by the moving object. In order to properly bind those locations corresponding to rigid motion, it has been demonstrated that motion sensors' output must be construed as a map of vectors under a Riemannian (non-Euclidean) geometry that is induced by the momentary image structure (Zhang and Wu, 1990). The Levi-Civita connection, which determines how vectors in nearby locations are to be compared (under the rule of parallel transport), has been derived and related to the image luminance function. The Riemann curvature is proven to be identically zero, indicating that vector comparison is well-defined (path-independent) within a region. This guarantees the mathematical feasibility of this differential geometric approach to perceptual binding. Also derived are the metric tensor, which is equal the product of the Hessian matrix of image luminance function, and the set of geodesics, which connect those points at which the first-order image luminance gradient are constant. This framework is applied to the problem of motion segregation in the random-dot kinematogram a la Braddick (1974), in which the segregation of a surface patch depends on an underlying 2-D rigid translation alone. A simple algorithm is provided to implement neural computation of the proposed geometry by using orientation and direction selective units (Zhang, 1995). Figural segregation is achieved through comparison and congruency of the motion vector field with families of orientation maps that are simultaneously calculated to represent the image geometry.

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